

DTIII 247

DK 551.511.32

Hydrodynamics and Nonlinear Instabilities

EDITED BY

CLAUDE GODRÈCHE

Centre d'Etudes de Saclay

PAUL MANNEVILLE

Ecole Polytechnique

322/3967

INSTITUT

FÜR METEOROLOGIE U. KLIMATOLOGIE

UNIVERSITÄT HANNOVER

HERRENHÄUSER STR. 2 - 30419 HANNOVER



CAMBRIDGE
UNIVERSITY PRESS

Contents

<i>Preface</i>	xiii
<i>Contributors</i>	xvi
Overview	1
<i>P. Manneville</i>	
1 An introduction to hydrodynamics	25
<i>B. Castaing</i>	
1 What is a fluid?	25
1.1 Introduction	25
1.2 Viscosity and Reynolds number	26
1.3 The basic equations	28
1.4 Momentum budget: two examples	29
1.5 Energy and entropy budget	32
1.6 The Navier–Stokes equation	34
1.7 Life at low Reynolds numbers	35
2 The mechanisms	40
2.1 Vorticity: diffusion, freezing, sources	40
2.2 Energy transfer in turbulence	44
2.3 Boundary layers and their separation	48
2.4 Compressibility	54
3 Flow measurement methods	57
3.1 Introduction	57
3.2 Hot wire anemometer	58
3.3 Doppler laser anemometer	61
4 Dimensional analysis	62
4.1 Buckingham's theorem	62
4.2 A simple example	63
4.3 A less simple case	64
4.4 Turbulent boundary layer	65
4.5 Barenblatt's second kind self-similarity	67

5 Modern approaches in turbulence	69
5.1 The Lorenz model	69
5.2 Statistical mechanics for turbulence	71
5.3 Developed turbulence	74
References	79
2 Hydrodynamic instabilities in open flows	81
<i>P. Huerre and M. Rossi</i>	
1 Introduction	81
1.1 An open flow example: the pipe flow experiment of Reynolds	84
1.2 A closed flow example: Taylor–Couette flow between rotating cylinders	90
2 Phenomenology of open flows	94
2.1 The mixing layer as a prototype of noise amplifier	96
2.2 The wake behind a bluff body as a prototype of hydrodynamic oscillator	104
2.3 Plane channel flow as a prototype of a viscous instability	111
3 Fundamental concepts	116
3.1 Some formal definitions	117
3.2 Linear instability concepts	119
3.3 Nonlinear instability concepts	145
4 Inviscid instabilities in parallel flows	152
4.1 Squire’s transformation	154
4.2 The two-dimensional stability problem: Rayleigh’s equation	155
4.3 Rayleigh’s inflection point criterion	158
4.4 Fjørtoft’s criterion	160
4.5 Jump conditions at an interface. Application to the vortex sheet	164
5 The spatial mixing layer	169
5.1 Linear instability of parallel mixing layers	169
5.2 Weakly non parallel WKB formulation	186
5.3 Secondary instabilities	193
6 The wake behind a bluff body	198
6.1 Linear instability of locally parallel wakes	200
6.2 Global instability concepts for spatially developing flows	207
6.3 Phase dynamics of wake patterns	220
7 Viscous instabilities in parallel flows	226
7.1 Squire’s transformation	229
7.2 The two-dimensional stability problem: the Orr–Sommerfeld equation	229
7.3 A first look at the instability mechanism: the energy equation	230
7.4 Heuristic analysis of the structure of two-dimensional Tollmien–Schlichting waves	232

8 Plane channel flow	243
8.1 Primary linear instability	243
8.2 Weakly nonlinear analysis	248
8.3 Finite-amplitude two-dimensional vortical states	256
8.4 Universal elliptical instability	266
8.5 Fundamental and subharmonic secondary instability routes	281
References	288
3 Asymptotic techniques in nonlinear problems: some illustrative examples	295
<i>V. Hakim</i>	
Introduction	295
1 Boundary layers and matched asymptotic expansions	295
1.1 An elementary example of a boundary layer; inner and outer expansions	297
1.2 Landau and Levich coating flow problem	300
2 Multiscale analysis and envelope equations	309
2.1 The period of the pendulum by the multiscale method	310
2.2 The nonlinear Schrödinger equation as an envelope equation for small amplitude gravity waves in deep water	312
2.3 Amplitude equation from a more general viewpoint	323
3 Fronts and localized states	325
3.1 Front between linearly stable states	325
3.2 Invasion of an unstable state by a stable state	338
4 Exponentially small effects and complex-plane boundary layers	346
4.1 Introduction	346
4.2 The example of the geometric model of interface motion	349
4.3 The viscous finger puzzle	359
4.4 Miscellaneous examples of exponential asymptotics in physical problems	370
References	382
4 Pattern forming instabilities	387
<i>S. Fauve</i>	
1 Introduction	387
1.1 Example: the Faraday instability	387
1.2 Analogy with phase transitions: amplitude equations	391
1.3 Long-wavelength neutral modes: phase dynamics	392
1.4 Localized nonlinear structures	393
2 Nonlinear oscillators	394
2.1 Van der Pol oscillator	395
2.2 Parametric oscillators	401

2.3	Frequency locking	405
3	Nonlinear waves in dispersive media	410
3.1	Evolution of a wave-packet	412
3.2	The side-band or Benjamin–Feir instability	417
3.3	Solitary waves	420
4	Cellular instabilities, a canonical example:	
	Rayleigh–Bénard convection	424
4.1	Rayleigh–Bénard convection	424
4.2	Linear stability analysis	429
4.3	Nonlinear saturation of the critical modes	432
5	Amplitude equations in dissipative systems	438
5.1	Stationary instability	439
5.2	Oscillatory instability	445
5.3	Parametric instability	447
5.4	Neutral modes at zero wavenumber.	
	Systems with Galilean invariance	449
5.5	Conserved order parameter	450
5.6	Conservative systems and dispersive instabilities	452
6	Secondary instabilities of cellular flows:	
	Eckhaus and zigzag instabilities	454
6.1	Broken symmetries and neutral modes	454
6.2	Phase dynamics	456
6.3	Eckhaus instability	457
6.4	The zigzag instability	469
7	Drift instabilities of cellular patterns	473
7.1	Introduction	473
7.2	A drift instability of stationary patterns	475
7.3	The drift instability of a parametrically excited standing wave	476
7.4	The drift bifurcation	478
7.5	Oscillatory phase modulation of periodic patterns	479
8	Nonlinear localized structures	481
8.1	Different types of nonlinear localized structures	481
8.2	Kink dynamics	484
8.3	Localized structures in the vicinity of a subcritical bifurcation	486
	References	489
5	An introduction to the instability of flames, shocks, and detonations	493
	<i>G. Joulin and P. Vidal</i>	
1	Introduction and overview	493
2	Basic equations	495
2.1	Conservation laws for reactive fluids	495

2.2 Weak forms	498
3 Subsonic versus supersonic traveling waves	499
3.1 The (p - V) plane	499
3.2 Various waves	500
3.3 Shocks, detonations, and deflagrations	501
4 Flames	503
4.1 Phenomenology	503
4.2 Minimal model and isobaric approximation	507
4.3 The basic eigenvalue problem	512
4.4 Jumps across the reaction layer	527
4.5 Diffusive instabilities	531
4.6 A conductive instability	540
4.7 Hydrodynamic instability	546
4.8 Body forces	568
4.9 Hydrodynamic influence of boundaries	577
4.10 Large-scale flow geometry	580
4.11 Prospects	585
5 Shock waves	592
5.1 Phenomenology	592
5.2 Shock formation	594
5.3 Majda and Rosales' model problem	612
5.4 D'yakov-Kontorovich's instabilities	614
5.5 Prospects	626
6 Detonations	627
6.1 Phenomenology	627
6.2 Chapman-Jouguet model and sonicity condition	631
6.3 Analogs of D'yakov-Kontorovich's instabilities	638
6.4 Brun's model for autonomous diverging waves	641
6.5 Zel'dovich-Von Neumann-Doering model	649
6.6 Chemistry-related instabilities	657
6.7 Recent results and prospects	663
References	667
<i>Index</i>	675